How To Tell Circles from Ellipses: Perceiving the Regularity of Simple Shapes

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Abstract Human observers achieve a surprising precision in many visual judgements, such as estimating relative position, colinearity and the regularity of shape. We measured the sensitivity in detecting shape deformations by presenting a square simultaneously with a rectangle of variable aspect ratio, or a circle with an ellipsoid. Weber fractions approach 3–5% and improve to approximately 1% when subjects are asked to tell which of the two objects was 'oriented more vertically', instead of identifying the square or circle. Contour position can be judged with a precision of 10–20 arc s, clearly in the hyperacuity range and also beyond the thresholds known for detecting differences in the curvature of comparable line segments. Our results suggest that detecting deformation in rectangles seems to rely on aspect ratio, whereas performance is improved for ellipsoids by a high sensitivity for changes in local curvature.

In everyday life human observers show extreme sensitivity to subtle distortions in the visual scene, such as pictures slightly out of balance and minute deformations of ideal shapes (Braitenberg 1984). Many visual judgements go far beyond the spatial resolution of the human visual system, the classical example for such hyperacuity being the detecting the misalignment of two line segments in a Vernier stimulus (Klein and Levi 1985; Morgan 1991). We analysed the limiting factors for detecting the distortion of a perfect circle or square, because the information that can be used to evaluate these simple shapes is restricted and well defined. To discriminate a perfect square from a rectangle with non-identical side lengths the visual system needs to estimate width and height and assess shape on the aspect ratio. Changes in local curvature could be used in addition to solve the corresponding task for a circle and ellipse. It has been known for some time that human observers and monkeys can detect 2–6% changes in the aspect ratio of ellipses (Laursen and Rasmussen 1975). Sensitivity is highest for shapes approaching perfect squares and circles, when subjects estimate the aspect ratio of rectangles or ellipsoids in the absence of absolute size cues (Regan and Hamstra 1992). This might indicate a special role for regular shapes or 'good form', as suggested by Gestalt psychology (Wertheimer 1923). By measuring thresholds for detecting deformed shapes, the purpose of our study was to compare directly the performance for squares and circles, and thus to assess the role of curvature information in the shape perception.

A square, or circle, was presented together with a rectangle, or ellipse, that was extended vertically or horizontally while keeping the area constant. The amount of deformation was varied by adjusting the radii (or side length in the case of rectangular shapes) in horizontal (a) and vertical (b) direction (see inset in Fig. 1). The average radius of the ellipse (or half side length of the rectangle, r), the horizontal separation between the two objects (d) and the presentation duration (T), were treated as stimulus parameters. A fixation cross was presented between the two objects, or free eye movements were allowed. Stimuli were presented on a monitor (Eizo T662, 1024×784 pixel resolution) driven by a digital stimulus generator (VSG 2/3, CRS) at a frame rate of 61 Hz. A blank screen was shown before and after the stimulus. The shapes were plotted as dark lines (minimum luminance 1.6 cd/m^2) with a triangular profile (four pixel half-width) on a bright back-

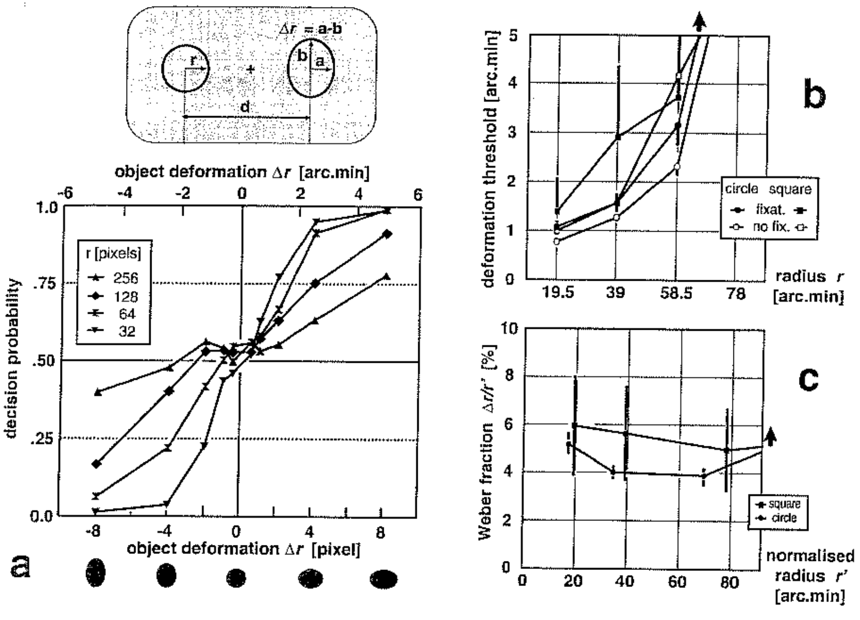

Fig. 1. a) Psychometric curves showing the proportion of decisions as a function of object deformation Δr (shapes *below abscissa*) for four different object sizes r (indicated by four different symbols; see *inset*). Data are pooled for circles and squares, and for fixation and free eye movement conditions. b) Thresholds of deformation detection, estimated by Probit analysis from psychometric curves, are plotted as function of object size r, for circles and squares (indicated by *corresponding symbols*) with fixation (*open symbols*) and free eye movements (*black symbols*). For the largest object some thresholds lie outside the deformation range tested, and therefore their estimates are unreliable. c) Weber fractions, calculated from the deformation thresholds as function of normalised radius r' (half square root of object area), for circle and square stimuli (different symbols, data from both fixation conditions pooled). Data from four subjects averaged, 320 decisions for each Δr and size; *error bars* SEM; *arrows* thresholds for the largest objects fall outside the plot range

ground (70 cd/m^2). The graded luminance profile allowed the line position to be defined with a precision beyond the monitor resolution by shifting the centre of the intensity distribution in sub-pixel steps (see Morgan and Watt 1982). The angular size of each screen pixel was 0.61 arc min at the default viewing distance of 200 cm.

Five subject with sight that was normal or corrected to normal (aged 23–45 years) took part in the experiments: one of the authors and four experienced observers naive to the purpose of the experiment. They were asked to decide which of the two simultaneously presented objects was the square/circle, compared to a horizontally or vertically elongated rectangle/ellipse. No feedback was provided. The position and orientation of the elongated object was varied in pseudo-random order, leading to equal numbers of horizontally or vertically oriented objects and perfect shapes on each side. The location of each object on the screen was varied randomly within the range of the maximum deformation to exclude absolute position cues. A set of stimuli with variable deformation $\Delta r = a\text{-}b$ was randomised in a method of constant stimuli. Stimulus parameters (r, T or d) were varied between blocks of tests, and circles and squares were tested in interleaved blocks with and without fixation target. The number of correct responses was converted into the number of decisions in which the more vertical object was considered to be the circle (see Fig. 1a), and deformation thresholds were calculated by Probit analysis (Finney 1962) from psychometric curves with data pooled for the two orientations.

The most obvious stimulus parameter affecting perceived shape distortions is the overall dimension of the stimulus. The size r of the objects was varied between blocks of 100 decisions in a range between 32 and 256 pixels, corresponding to 0.33° and 2.60° (separation $d = 512$ pixel, duration $T = 1$ s). Within each block the deformation Δr was varied between 0.5 and 8 pixel to generate horizontally ($\Delta r > 0$) and vertically ($\Delta r < 0$) elongated shapes. The slopes of the average psychometric curves plotted in Fig. 1a increase with decreasing r, without any obvious asymmetry between vertical and horizontal deformations. Deformation thresholds, calculated from the number of correct decisions for each deformation Δr, are plotted separately in Fig. 1b for the two viewing conditions and for both shapes. Apart from the strong increase in thresholds with object size r, there seems to be a trend that free eye movements improve the performance, and that deformations are better detected in circles than in squares. The minimum threshold of 1.22 pixel for the smallest circle with fixation corresponds to a contour displacement of 0.7 arc min, close to the human resolution limit. Thresholds plotted as Weber fractions (Fig. 1c) show only little variation for the three small objects. Because the distance of the object border to its centre varies for the square, and its average differs from that of a circle with a diameter equal to the side length of the square, Weber fractions are shown as function of the half square root of object area, r'. Even after this re-scaling of object size, the performance for circles, slightly below 4%, tends to be better than that for squares which stay around 5% under the optimum conditions tested here. This basic result holds for a range of horizontal separations between the objects between $d = 160$ and 340 pixel, and in a first approximation also for a range of presentation durations between $T = 250$ and 4000 ms (data not shown).

The horizontal and vertical extent of an object could either be 'measured' from a static retinal projection, or the motor signal of scanning eye movements could be used to estimate the distance between image points. Although one may question the precision of eye movements, there are hints in our data to support such a view. In Fig. 2a deformation thresholds from all circle/ellipse experiments are plotted as a scatter diagram, with each point representing a pair of thresholds measured for the same stimulus with fixation spot and with free eye movements, respectively. The majority of data points above the diagonal show that thresholds are increased under fixation conditions (the same pattern of results is found for squares/rectangles). Although this indicates that allowing eye movements can improve performance, one cannot conclude that they are used to estimate object dimensions as such. The informal observation that subjects alternate their gaze between the two objects could be interpreted as involvement of eye movements in the comparison process.

Our subjects frequently reported that they clearly could discriminate the two objects in our stimuli but were not sure which was the perfect circle (or square), suggesting that the reference to a memory representation of the regular shape impairs performance. In a control experiment, using exactly the same stimuli, the subjects therefore were not asked to identify the circle or square shape, but they had to decide which of the two objects is oriented more vertically. This implicit discrimination task leads to considerably lower thresholds than the explicit task to identify the regular shape. To assess performance limits, two subjects were given extensive practice in a large number of sessions. When thresholds approached an asymptotic plateau, subjects were tested with circles and squares in ten sessions, each of which comprised one square and one circle threshold measurement, in counterbalanced order. The small deformations necessary for these highly trained subjects were made possible by extending the viewing distance to 4 m and using a set of smaller deformations, now ranging from 0.25 to 4 pixel. Figure 2b presents the results from this experiment as a scatter diagram of thresholds for rectangles plotted against those for ellipses. From the majority of the data points lying above the diagonal it is obvious that thresholds are generally higher for the rectangular shapes. The difference of the thresholds for the two conditions are highly significant for both subjects (JMZ: 13.2 ± 0.80 arc s and 20.6 ± 2.4 arc min for circles and squares, respectively, $P < 0.02$, Student's t test; TIM: 15.2 ± 1.4 and 21.4 ± 1.5, $P < 0.01$). The range of thresholds measured in this experiment, between 10 and 20 arc s, is clearly below the resolution limits of the human visual system and compare well with the limits for other hyperacuity tasks (Westheimer 1981).

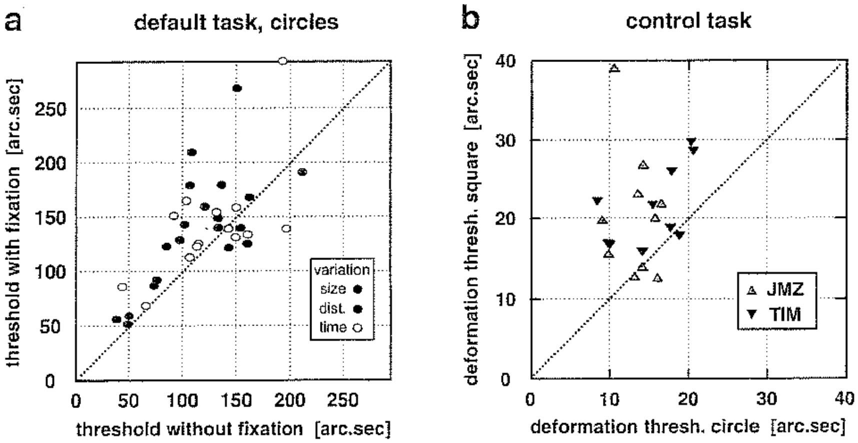

Fig. 2. a) Scatter diagrams plotting the thresholds for the detection of shape deformation in circles with free eye movements (*abscissa*) and fixation of a central target (*ordinate*) against each other for a variety of conditions (variations in size, separation, stimulus duration, indicated by different symbols). *Each data point* one pair of thresholds measured in an individual subject in two tests with and without fixation but with otherwise identical stimulus parameters. b) Scatter diagrams showing the relationship between the detection of shape deformation in squares (*ordinate*) and circles (*abscissa*) for the control task in which the subjects had to identify the more vertical object. Ten pairs of deformation thresholds from each of two subjects (different symbols). Performance was optimised by using small circles ($r = 64$ pixel), medium inspection times ($T = 1$ s) and free viewing. The overwhelming majority of data points lie above the diagonal, indicating a higher sensitivity for circles

Our experiments confirm that human observers can detect deformations of regular shapes, such as a perfect circle or square, with a high precision, reaching Weber fractions below 5% at optimum conditions for identifying circles or squares. Deformation thresholds decrease into the hyperacuity range if memory-based identification of the regular shape is not required. What is the mechanism underlying this impressive discrimination performance? If the detection of shape deformation were exclusively based on the aspect ratio of the objects, one would expect a similar performance for squares and circles. Squares might even be expected to be superior to circles because the dimensions should be much easier to estimate along the extended horizontal and vertical lines of a square than relying on the widest and most narrow diameters in an ellipse. However, our observers performed much better with circles than with squares, and a similar but weak trend may be found in the data of Regan and Hamstra (1992). It is of particular importance that our subjects had reached optimum performance after extensive practice in both tasks, and were tested in alternation with the two shapes within each session. The advantage of circles over squares indicates that curvature plays an important role in solving our detection task. Our thresholds for detecting deformations convert into differences in curvature in the range of 0.00037 and 0.00017 arc min (for comparable default and control conditions). These values are much lower than those described in the literature for isolated circle segments of similar geometry (Foster et al. 1993), which lie above 0.002 arc min at curvatures of 0.01–0.02 arc min. This suggests that curvature information is integrated along the contours of the circle, thus improving performance over isolated line segments. Similar conclusions about the importance of global processing were drawn from rather different experiments involving orientation discrimination of symmetrical patterns such as ellipses (Li and Westheimer 1997) and the detection and identification of radial frequency patterns featuring variations in local curvature (Wilson et al. 1997; Wilkinson et al. 1998).

It is well known that two positions in the retinal image can be compared with much higher accuracy than that provided by the spatial resolution of the visual system (Klein and Levi 1985; Morgan 1991; Wilson 1986). The processing of curvature, requiring the comparison of relative position of at least three points in space, seems to reach similar limits (Watt and Andrews 1982; Whitaker et al. 1993; Kramer and Fahle 1996). The results from our simple stimulus condition, so far only tested for a limited range of stimulus parameters, indicate that sensitivity in shape discrimination may rely critically on changes in local curvature along the circumference of an ellipse. This indicates a specific role for the integration of relative position information (see Field et al. 1993; Wang and Levi 1994; Mussap and Levi 1996) in highly accurate shape perception.

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Braitenberg V (1984) Vehicles. Experiments in synthetic psychology. MIT Press, Cambridge

Field DJ, Hayes A, Hess RF (1993) Contour integration by the human visual system: evidence for a local "association field." Vision Res 33:173

Finney DJ (1962) Probit analysis. Cambridge University Press, Cambridge

Foster DH, Simmons DR, Cook MJ (1993) The cue for contour-curvature discrimination. Vision Res 33:329

Klein SA, Levi DM (1985) Hyperacuity thresholds of 1 sec: theoretical predictions and empirical validitation. J Opt Soc Am A 2:1170

Kramer D, Fahle, M (1996) A simple mechanism for detecting low curvatures. Vision Res 36:1411

Laursen AM, Rasmussen JB (1975) Circle-ellipse discrimination in man and monkey. Vision Res 15:173

Li W, Westheimer G (1997) Human discrimination of the implicit orientation of simple symmetrical patterns. Vision Res 37:565

Morgan MJ (1991) Hyperacuity. In: Regan DM (ed) Vision and visual dysfunction 10. Spatial vision. Macmillan, Houndsmill, p 87

Morgan MJ, Watt RJ (1982) Mechanisms of interpolation in human spatial vision. Nature 299:553

Mussap AJ, Levi DM (1996) Spatial properties of filters underlying Vernier acuity revealed by masking: evidence for collator mechanisms. Vision Res 36:2459

Regan D, Hamstra SJ (1992) Shape discrimination and the judgement of perfect symmetry: dissociation of shape from size. Vision Res 32:1845

Wang H, Levi DM (1994) Spatial integration in position acuity. Vision Res 34:2859

Watt RJ, Andrews DP (1982) Contour curvature analysis: hyperacuities in the discrimination of detailed shape. Vision Res 22:449

Wertheimer M (1923) Untersuchungen zur Lehre von der Gestalt. Psychol Forschung 301

Westheimer G (1981) Visual hyperacuity. In: Autrum H, Ottoson D, Perl ER, Schmidt RF (eds) Progress in sensory physiology. Springer, Berlin Heidelberg New York, p 1

Whitaker D, Latham K, Mäkelä P, Rovamo J (1993) Detection and discrimination of curvature in foveal and peripheral vision. Vision Res 33:2215

Wilkinson F, Wilson HR, Habak C (1998) Detection and recognition of radial frequency patterns. Vision Res 38:3555

Wilson HR (1986) Responses of spatial mechanisms can explain hyperacuity. Vision Res 26:453

Wilson HR, Wilkinson F, Asaad W (1997) Concentric orientation summation in human form vision. Vision Res 37:2325